

Estimating Strategic Complementarities in Credit Unions' Outsourcing Decisions

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Abstract: We examine the choice between internal or external provision of information technology (“IT”) services for US credit unions. Empirically, the likelihood that a credit union outsources its IT is increasing in the number of other credit unions in the same geographic market who outsource. We estimate a game theoretic model in which the simultaneous outsourcing decisions of the other credit unions in the market are contained as an argument in the relative costs associated with outsourcing. We adapt methods for estimating entry models with multiple equilibria to a setting with strategic complementarities, where multiple equilibria are endemic. Since our approach does not require one to select a particular equilibrium when there is more than one, we can assess how firms coordinate to particular equilibria in different types of markets. Our main empirical finding is that the likelihood of coordination failure is U-shaped in the number of firms in the market, reaching its minimum at five firms.

JEL Codes: L1, C5, C7

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I. Introduction

We examine the choice between internal or external provision of information technology (“IT”) services for US credit unions. Credit unions may maintain their own systems for tracking loans and deposit accounts or they may choose to outsource these systems from external providers. Empirically, the likelihood that a credit union outsources its system is increasing in the number of other credit unions in the same geographic market that outsource. This regularity may be due to characteristics common to the credit unions in close proximity or characteristics of the market itself that make it more or less likely that credit unions make the same technology choice. It may also be due to complementarities whereby one credit union’s decision to outsource affects the relative costs associated with outsourcing for the other credit unions in the market.

Anecdotal evidence suggests that complementarities may result from communication among credit unions through local associations and state level leagues and which is further facilitated by credit unions’ non-profit status and lack of significant competitive overlap. Even if complementarities exist, it is nonetheless possible that a lack of coordination among participants in the market can lead to suboptimal outcomes. In this paper, we attempt to measure the size of strategic complementarities in the cost function for IT and to assess the degree of coordination such complementarities engender.

To accomplish this, we estimate a game theoretic model in which credit unions outsource if doing so achieves a cost savings relative to maintaining their own systems. The decision to outsource is modeled as a game in which the simultaneous outsourcing decisions of the other credit unions in the market are contained as an argument in the relative costs associated with outsourcing. The game gives rise to an econometric model

that is “incomplete.” In particular, the existence of multiple pure strategy Nash equilibria (“PSNE”) implies that the likelihood associated with those outcomes is not well-defined, precluding standard maximum likelihood estimation.

Researchers estimating payoff functions in these types of games have confronted complications caused by existence of multiple equilibrium outcomes, dating back at least to the work of Bresnahan and Reiss (1990). In recent papers, Tamer (2002), Ciliberto and Tamer (2004) (“CT”), and Andrews, Berry and Jia (2004) (“ABJ”) have made significant progress on this problem. In particular, these papers have suggested methods for estimating payoff parameters assuming only that observed outcomes represent *a* PSNE, without specifying any selection rule in cases where the model is consistent with multiple PSNE. This literature has been inspired by the study of entry games and other games in which players’ actions are strategic substitutes in the sense that a given agent’s payoff is decreasing in the number of other players taking the same action.

In contrast, our study of credit union outsourcing is an example of a game with strategic complementarities where a given agent’s payoff is increasing in the number of other players who take the same action. In this case, multiple PSNE are also endemic; though the problems arising from multiplicity in these games have received significantly less attention, in large part because the structural modeling of social interactions and network externalities (two leading examples involving complementarities among agents’ actions) has been a relatively recent phenomenon.

To estimate the extent of complementarities in credit unions’ outsourcing decisions, we adapt the intuition in CT and propose a method for estimating payoff parameters that places no restrictions on which outcome obtains when the model is consistent with

multiple PSNE. We provide an algorithm to solve for the set of PSNE that is easy to implement and whose computational burden increases linearly, rather than exponentially, in the number of players.¹ Because we are able to overcome the dimensionality problem, we do not need to restrict the outcome space (as in CT and ABJ) nor do we need to observe the choices of the same firms in every (or indeed any) market. We also provide conditions under which the parameters are point (rather than set) identified.

Using these estimates, we are then able to assess the how the degree of coordination among credit unions varies with the size of the market (i.e., the number of credit unions). We find that the probability that the observed outcome is pareto dominated by another outcome, when the observed outcome is consistent with multiple PSNE, is U-shaped reaching its minimum at five credit unions.

In addition to the methodological contributions, understanding how other firms' decisions affect the payoffs associated with outsourcing is of broader interest as the practice of procuring intermediate services outside of the firm is gaining in prevalence in the financial services industry and the US economy as a whole.

The remainder of the paper is organized as follows. Section 2 presents background on credit unions, presents the data and discusses potential complementarities in credit unions' outsourcing decisions. Section 3 presents our theoretical model and characterizes the set of PSNE for this game. Section 4 presents the econometric framework and estimation strategy. Section 5 presents the results and section 6 concludes.

¹ The methods proposed to deal with multiplicity in this context suffer from a version of the "curse of dimensionality" where adding players imposes a substantial computational burden. Other researchers have confronted this issue by making ancillary assumptions to restrict the dimensionality of the outcome space, though this approach essentially ignores useful information.

II. Background

In this section we discuss a credit union's choice between providing IT in-house or outsourcing the IT function. We begin by considering the unique nature of credit unions and then contrast the two technologies. We present some summary statistics along with a probit regression that documents the relationship between a credit union's outsourcing decisions and the outsourcing decisions of other credit unions in the local market.

Credit Unions and Outsourcing

Credit unions are non-profit, cooperative financial institutions owned and run by their members. In 2003, over 80 million people were members of the approximately 9,700 credit unions in the United States. The institutions held roughly \$533 billion in deposits which accounts for nearly 10% of total consumer savings. Critically for our analysis, each credit union must define a field of membership, defining what customers the credit union can serve. This grouping can be based on residence in a given community, employment with a given employer or membership in a given organization or association. While a particular consumer may possibly be eligible for membership in more than one credit union, the membership requirement limits direct competition between these organizations. As a result, credit unions are more likely to share information than direct competitors would be. This is evident in the existence of credit union leagues in every state where member institutions share news and information as well as in the formation of credit union service organizations (CUSOs).

Information technology is crucial in the operation of credit unions, as it is throughout the financial services industry. The central role of information technology in the operation of financial services firms is often credited with fostering the increasing dominance of larger financial institutions most able to capture the increasing returns to scale inherent in the technology. An alternative organization of economic activity, however, has large outside suppliers of these services capturing the same returns to scale by aggregating the needs of smaller individual firms. The same information and communication technologies that may aid consolidation may also foster the existence of smaller financial services firms and to the evolution of an industry that specializes in providing IT services to financial institutions.

As of 2003, almost all credit unions have moved from managing their accounts on paper to some form of computerization. However, credit unions vary substantially in how they structure their IT systems, utilizing both forms of organization just discussed. While many credit unions choose to manage and store information in-house using software and hardware that is typically purchased, a substantial fraction hire an external firm to manage and store information outside of the credit union.

For credit unions that choose to manage their IT in-house, the primary costs relate to the hardware and software that need to be purchased (or developed) and maintained, in addition to ongoing labor costs for the personnel that manage the systems. Hardware costs involve a fixed upfront investment while software usually requires some upfront costs and then some ongoing licensing fees. The size and quality of the local labor market will affect the labor cost associated with keeping IT in-house.

In contrast, outsourcing IT requires smaller outlays for hardware and for an internal IT staff. Originally, computers at a credit union that outsourced their IT were dumb terminals connected point to point by dedicated telephone lines to the service provider's computers. More recently, the credit union need only purchase standard PCs and the ability to connect to the Internet. In either case, the hardware costs are far less than running an internal system. The labor costs are also substantially lower under this option since, by definition, the main hardware and data infrastructure reside at the service provider. For example, security, hardware and software maintenance, and upgrades at the data center are not direct expenses of the credit union, but rather of the outsourcer.

For these services, the credit union usually pays the service provider an initial fee and then an ongoing fee, related to the number of customers that they have. The initial fee is often less than an initial licensing fee for software; however, the ongoing fee may be higher. Conversations with industry participants indicate that on average the total fees paid to a service provider are about 30% higher than the fees for licensing comparable software. Presumably for those firms that outsource, the savings on hardware and labor as well as any net benefits outweigh this cost difference.

Discussions with some of the larger providers of credit union IT services indicate that the largest non-pecuniary difference between these two options rests in flexibility. In terms of managing IT, this includes losing the ability to control the timing of upgrades, or to use some third party software. For operations, this may include the inability to customize reports or to query data in particular ways. Outsourcing may even limit the products a credit union could offer to those supported by the service provider. However, given the economies of scale inherent in IT, it is also possible that procuring a new

service (such as payment card processing) from an outside firm is cost effective while a given credit union using their own IT would not find it worthwhile. In total, the relative pecuniary and non-pecuniary costs and benefits of each option determine what choice the firm will ultimately make.

A conversation with Leslie M. Muma, President and CEO of Fiserv provided some interesting insight into why strategic complementarities may and may not exist in the outsourcing decision. Mr. Muma suggested that (1) the number of firms in a given geographic market who choose to outsource their IT does not affect the costs associated with providing outsourcing services to any firm in that market; and (2) credit unions are much more likely to exhibit geographic clustering in their outsourcing decisions than are commercial banks.² Thus, potential spillovers that would reduce the costs associated with a given technology appear to take the form of credit unions sharing information about how to use each technology more efficiently. Indeed, further anecdotal evidence suggests that it would not be uncommon for one credit union to contact another credit union using the same IT to discuss problems as they arise. Furthermore, there is a sufficient degree of flexibility in the outsourcing option, while less than that associated with internal IT provision, so that consultation among credit unions that outsource their IT does take place regarding various day to day IT-related operations.

Data

² This conversation took place at the conference on Bank Structure and Competition in Chicago, IL on Thursday, May 6, 2004.

The primary data source for this study is the Second Quarter 2003 Call Report from the National Credit Union Administration (NCUA). Besides reporting financial data, credit unions are asked about the information technology that they employ. In particular, the following question is asked:

Indicate in the box at the right the number of the statement below which best describes the system the credit union uses to maintain its loan and share records:

- | | |
|---------------------------------|-----------------------------------|
| 1=Manual System (No Automation) | 2=Vendor Supplied In-House System |
| 3=Vendor On-Line Service Bureau | 4=CU Developed In-House System |
| 5=Other | |

We classify all credit unions using a service bureau (choice 3) as outsourcing while those using any other systems as using in-house software. Of the 9615 credit unions that filed in June 2003, the 232 still using manual systems are dropped from the sample.

Our model measures strategic complementarities among firms using in-house software as well as among firms that outsource. As described above, the presence of these effects for those using in-house software results primarily from information sharing among firms. While only 1.3% of the universe uses self-developed software and another 1.7% is listed as ‘other’, grouping these with those using in-house software may bias our results away from finding evidence of network effects among the credit unions using in-house systems. This effect may be small to begin with, given that even those using vendor supplied in-house software may be using systems from different vendors.

In order to estimate the model, we restrict the data to credit unions in markets with 16 or fewer credit unions. This limits the sample to 3570 credit unions in 1101 markets. On average, 24.8% of these outsource their IT. Table 1 shows this breakdown by market, displaying the actual distribution of market outcomes given the number of firms in the market. For example, both firms outsource in 8% of the two-firm markets

that we observe, while neither firm outsources in 61% of the two-firm markets that we observe.

The independent variables, summarized in Table 2, include two credit union specific and two market level variables that are assumed to influence the relative costs of outsourcing. The credit union variables include the log of assets and the number of branches for the given firm. Many IT decisions vary with the size of the firm and using assets as a proxy for size is the logical choice in this case. To handle the large skew in this variable, the minimum is only \$83,000 while the maximum in the sample is over \$2.5 billion, the log of assets is used in the estimation. The branch variable captures the variation in costs of implementing and maintaining either an in-house or outsourced system over a larger number of locations. A priori, it is not clear however whether the relative costs of outsourcing will be larger or smaller for credit unions with larger branch networks.

Following conventional practice, a credit union's market is defined as the MSA or non-MSA county in which it is located. Given this definition, the market level variables that we use include a dummy for non-MSA markets and the overall population of the market. Rural markets are likely to be more distant for outsourcers and therefore have a higher relative cost (and lower tendency) to employ that strategy. These markets make up 82% of the markets in the sample but contain only 55% of the credit unions. Population is a proxy for the availability of local IT services and we expect credit unions in larger markets to be less likely to outsource.

Probit Results

In order to describe the data, we first estimated simple probits and linear probability models. The results of these estimations are in Table 3. In each case, the probability of outsourcing is the dependent variable. The regressors include the size of the firm, the size of the branch network, the population and classification of its market as well as the number of other firms who do and do not outsource their IT systems.

The first column shows the coefficient estimates from the probit results while the second translates those into marginal effects. As expected, larger firms are more likely to outsource while rural firms, those with large branch networks and those in more populated markets are less likely. With the exception of the rural dummy, each of these coefficients is statistically significant.

The variables designed to measure strategic complementarities are also significant and show strong evidence that these effects exist. Conditional on the number of credit unions in the market, it is important to note that an additional firm outsourcing implies one fewer using in-house software. Thus, the effect of having an additional outsourcer is the difference between the first two coefficients in the table. In the basic probit results, this implies that having an additional firm outsourcing increases the probability that a given firm will outsource by over 5.5%, evaluated at the mean of the explanatory variables.

Bias may arise in the probit model for two related reasons. First, there may be omitted variables affecting the choices of firms that are correlated within a given market. If there is any interdependency amongst firms' decisions, these omitted variables will cause correlation between the unobservable and the number of other firms who

outsource, resulting in biased estimates. A second source of bias derives from model misspecification. If the model admits multiple equilibria then the likelihood can not be described using a probit model. We attempt to address the first of these concerns by estimating an instrumental variables linear probability model which is presented in the last column of Table 3. (Both concerns are addressed in the structural model.) In these estimates, the number of firms who outsource (insource) is instrumented by the sums and means of the two credit union specific variables for other firms in the market. This approach is suggested by Manski (1993), and has been used by Gowrisankaran and Stavins (2003) to estimate network effects. Column 2 presents the non-instrumented version of the linear probability model for comparison.

Both linear probability models show results similar to the probit. The effects for assets and branches are smaller in these models as is the effect of population. Interestingly, the complementarities in the linear probability model closely mirror those in the probit. Accounting for the endogeneity of these variables with the instruments reduces these effects somewhat, particularly for the effect of other firms' in-sourcing decisions.

III. Theoretical Model

In this section, we present our basic theoretical model that captures the interdependence of credit unions' outsourcing decisions in equilibrium. We explore characteristics common to all PSNE and are able to place important restrictions on the set of PSNE. These restrictions inform our estimation strategy which is discussed in Section 4. While we borrow from the entry literature in our estimation approach, important

differences between entry games and our game allow us to extend the existing econometric methods that have been applied to entry models.

Let $y_j = 1$ if firm j outsources its IT, $y_j = 0$ if firm j provides IT internally, and let Y be the vector that captures all firms' IT decisions. The number of credit in each market, N_m , is assumed to be exogenously given and, in our sample, it varies across markets.

We assume that each credit union's IT costs take the following form:

$$\mathbf{3.1} \quad \phi_i(Y) = \left[X_i \beta_1 + g(K^{-i}) + \varepsilon_{1,i} \right] y_i + \left[X_i \beta_0 + h(N_m - K^{-i} - 1) + \varepsilon_{0,i} \right] (1 - y_i)$$

where X_i is a vector of exogenous covariates that shift IT costs for each firm depending on whether the firm outsources its IT or provides it internally. $K^i = \sum_{j \neq i} y_j$, is the number

of credit unions in the market, other than i , that outsource their IT. The functions $g(K^{-i})$ and $h(N_m - K^{-i} - 1)$ capture the effect on i 's costs of the number of other credit unions in the market who choose to outsource and provide IT internally, respectively.

Finally, $\varepsilon_{1,i}$ and $\varepsilon_{0,i}$ are firm-specific cost shifters that are unobserved to the econometrician.

We can now define the *incremental* cost of outsourcing as:

$$\mathbf{3.2} \quad c_i(Y) = X_i (\beta_1 - \beta_0) + g(K^{-i}) - h(N_m - K^{-i} - 1) + (\varepsilon_{1,i} - \varepsilon_{0,i})$$

Combining terms, we can rewrite **3.2** as follows:

$$\mathbf{3.3} \quad c_i(Y) = X_i \beta + f(K^{-i}) + \varepsilon_i$$

where $\beta = \beta_1 - \beta_0$, $f(K^{-i}) = g(K^{-i}) - h(N_m - K^{-i} - 1)$, and $\varepsilon_i = \varepsilon_{1,i} - \varepsilon_{0,i}$.

A firm will choose to outsource, i.e., choose $y_i = 1$, if $c_i < 0$, or

$$3.4 \quad \varepsilon_i < -X_i\beta - f(K^{-i})$$

A PSNE of this game is a profile $Y = (y_1, \dots, y_{N_m})$ such that

$$3.5 \quad X_i\beta + f(K^{-i}) + \varepsilon_i < 0 \Leftrightarrow y_i = 1^3$$

i.e., all firms that outsource achieve a cost savings, and that those who do not outsource would not achieve a cost savings if they chose to do so.

At this point, it is useful to discuss the more important assumptions embedded in our specification of the incremental costs of outsourcing in **3.3** as well as the additional assumption of strategic complementarity. These assumptions are:

Assumption 1: X_i and N_m orthogonal to ε_i

Assumption 2a: $g(\bullet)$ and $h(\bullet)$ depend on only K^{-i} (as opposed to Y)

Assumption 2b: $g(\bullet)$ and $h(\bullet)$ do not vary across firms⁴

Assumption 3: $g(\bullet)$ and $h(\bullet)$ are weakly decreasing.

XX FILL IN DISCUSSION HERE XX

Properties of the Set of Pure Strategy Nash Equilibria

In general, the game we study admits multiple PSNE. As discussed below, this poses significant difficulties for estimating the model as we have specified it. We are able, however, to place some restrictions on the set of PSNE outcomes that will allow us

³ This is not precisely correct since a firm may either provide IT internally or outsource if $c_i=0$. We ignore this in our presentation here since the distributional assumptions (namely that the error term has continuous support) we use to estimate the model imply that $c_i=0$ happens with zero probability.

⁴ Note that g and f could be permitted to vary across firms with no difference in our theoretical results.

to make use of recent techniques developed in Tamer, ABJ and CT for estimating models with multiple PSNE. In particular, we show that there are at most $\frac{N_m}{2} + 1$ PSNE in our game and that it is sufficient to search over only $N_m + 1$ outsourcing configurations in order to fully characterize the set of PSNE. This is in contrast to the 2^{N_m} possible PSNE in the entry game studied by CT, whose estimator we adapt. This result stems from our assumptions that the incremental costs of outsourcing from equation 3.3 are weakly decreasing in the number of firms who outsource, and depend only on the number, not the identities, of the other firms who outsource.

We can now establish Proposition 1 which reduces the set of possible PSNE from 2^{N_m} to $N_m + 1$.

Proposition 1: Any PSNE in which N firms choose to outsource, will always involve the same N firms choosing to outsource.

Proofs of all propositions are presented in an appendix.

Proposition 1 implies that there are at most $N_m + 1$ PSNE, where N_m is the number of firms in market m .⁵ This observation significantly simplifies the search for possible PSNE. Proposition 2 shows that outsourcing decisions must follow the ordering of firms' incremental outsourcing costs, and that (similar to Berry 1992) this ordering does not depend on the outsourcing decisions of other firms in the market.

Proposition 2: The N firms with the *lowest* incremental costs of outsourcing will all choose to outsource in any PSNE in which N firms outsource.

⁵ The additional PSNE is the outcome in which no firm outsources.

The ability to order the firms' outsourcing costs for any strategic interaction, allows us to establish Proposition 3, which implies that there are at most $\frac{N}{2} + 1$ PSNE.

Proposition 3: If there is a PSNE in which N firms choose to outsource, a PSNE in which $N + 1$ firms choose to outsource does not exist.

Propositions 1-3 place significant restrictions on the set of PSNE of our game. We will use these restrictions below to modify the estimators proposed by Tamer, ABJ and CT so that they can be applied in a parsimonious way to the estimation of strategic complementarities.

IV. Econometric Model

The purpose of this section of the paper is to outline our estimation strategy. Before doing so, however, we discuss how the previous literature has approached the estimation of discrete games with multiple PSNE in the context of entry games, network effects, and social interactions. After introducing the estimator, we discuss our algorithm for identifying the entire set of PSNE. Finally, we discuss identification of the model's parameters.

Previous studies of games with multiple PSNE

The nature of the game we consider, and indeed any game with strategic complementarities, precludes straightforward maximum likelihood estimation due to the fact that the likelihood will not be well-defined if the game admits multiple equilibria. Previous work on network effects and social interactions has confronted the issue of multiplicity either by making assumptions that guarantee a unique equilibrium or by

assuming a particular selection rule. We employ a different tact. In particular, we make the minimal assumption that each market is in a PSNE outsourcing configuration, without specifying any selection rule. In general, our minimal assumptions may not be enough to identify the parameters of interest, but they are capable of identifying the set of parameters that are consistent with the game as we specify it below. We do, however, derive sufficient conditions for point identification of the model's parameters.

We approach the estimation of our model in a manner that is conceptually similar to entry games studied by Bresnahan and Reiss (1990), Berry (1992), Tamer (2002), and CT. Bresnahan and Reiss (1991) show that for rich enough error structures, their entry game will always admit multiple PSNE (as indeed is the case in our setup). In the entry context, actions by other firms (namely whether to enter or not) are strategic substitutes. In an entry game with two firms, for example, it will be possible to find values of the unobservables such that either firm, but not both, may enter in equilibrium. To the extent that the decision to outsource is associated with network effects, firms' actions are strategic complements. In the example above, for some values of the exogenous variables, it will be possible to find values of the unobservables such that either both firms outsource or neither firm outsources. In the presence of multiple PSNE, therefore, the likelihood of a given profile of strategic choices is not well defined.

While a number of solutions to the problem of multiple equilibria have been suggested for the case of strategic substitutes, most are not appropriate in our context. Bresnahan and Reiss (1990) and Berry (1992) suggest focusing on the total number of entrants (which, for many entry models is constant across PSNE), rather than their

identity.⁶ The number of firms who choose to outsource, however, is not constant across PSNE since the outsourcing decisions of different firms are strategic complements.

The study of multiplicity in games with complementarities has been motivated by increased interest in social interactions and network externalities. Brock and Durlauf (2004) consider the identification of discrete choice models with social interactions. They avoid the issue of multiplicity by assuming that individual preferences are a function of one's *beliefs*, which are self-confirming in equilibrium, about how other members of the group will behave. They recognize that the assumption will be problematic for smaller sized groups which we observe if one considers the credit unions in a market to be a "group." Furthermore, since the complementarities in our context relate to the sharing of information about a particular IT option, it is the actual IT decisions of the other credit unions that affect a given firm's costs, not beliefs about what these decisions will be. The literature on network effects has also confronted the issue of multiplicity. Sweeting (2004) studies a game with imperfect information that admits only two possible equilibria and shows that the multiplicity facilitates identification of firms' incentive to coordinate. Augerau, Greenstein and Rysman (2004) assume a unique equilibrium during estimation and then check to see if there are other equilibria ex-post. Akerberg and Gowrisankaran (2003) structurally estimate network effects in the ACH Banking Industry by expressing the likelihood as a weighted average of the likelihoods associated with the Pareto worst and best equilibria in each market. The probability that either equilibrium is the one that is actually played is estimated assuming a constant selection rule.

⁶ Davis (2003) uses a similar approach, focusing on the total level of output (for example the number of retail outlets in a market) as opposed to the output of each firm.

We adapt and extend methods that have been used to accommodate the existence of multiple PSNE in entry models. In particular, papers by Tamer (2003), CT, and ABJ have proposed methods for estimating models that have multiple PSNE without imposing any structure on which equilibrium is selected. Tamer (2003) shows that, while the likelihood may not be well defined, one can derive informative bounds on the likelihood that can be used as the basis of a modified maximum likelihood (MML) estimator. CT and ABJ use similar approaches to form a modified minimum distance (MMD) estimators that can be used in settings with more than two potential entrants.

An alternative approach that we do not choose to adopt is that suggested in Bajari, Hong and Ryan (2004) who consider dynamic entry games in an imperfect informational setting. They identify equilibrium selection rules from exclusion restrictions on variables that affect equilibrium selection but not firms' objective functions. Our strategy is motivated by our interest in characterizing the degree of coordination among firms. We opt for an estimation approach that requires no assumptions about equilibrium selection, but afterward allows us to analyze certain efficiency aspects of the selected PSNE when multiple PSNE exist.

Adaptation of the MMD estimator

Our general approach, which follows from CT, is to bound the probability of observing certain outcomes, and to find the set of parameters for which the “true” probabilities of those outcomes lie within the bounds given by our model. In the previous section, we showed that under our assumptions the dimensionality of the space of potential equilibrium outcomes is significantly reduced. In this section, we show that the

structure of the set of PSNE for this game can be completely characterized using a simple algorithm. The large number of potential outcomes associated with games where actions are strategic substitutes makes implementation of the MMD estimator almost impossible.⁷ CT, for example, collapse the number of potential entrants to five by combining similar types of firms into a single firm for this very reason. Our algorithm is easy to implement for any number of firms. Furthermore, we do not need to observe the same firms in every (or even *any*) market.

Tamer (2003) considers a model with two potential entrants, where, for certain values of the unobservables, either firm may enter as a monopolist, but not both. Intuitively, bounds on the likelihood are obtained by assuming different selection rules for the region of the errors which yields multiple PSNE. The likelihood of $(1,0)$, where (y_1, y_2) entry by firm i corresponds to $y_i = 1$, can not be larger than the likelihood that $(1,0)$ is a PSNE. On the other hand, the likelihood of $(1,0)$ must be at least as large as the likelihood that $(1,0)$ is the *unique* PSNE. That is, the upper bound, $\bar{P}(1,0)$ is obtained by assuming that $(1,0)$ always obtains in the region in which both $(1,0)$ and $(0,1)$ are Nash equilibrium outcomes. The lower bound, $\underline{P}(1,0)$, is obtained by assuming that $(1,0)$ never obtains in the region in which both $(1,0)$ and $(0,1)$ are Nash equilibrium outcomes.⁸

Tamer (2002) and CT extend this intuition to the case of several potential, N_m , entrants. They propose a modified minimum distance (MMD) estimator that requires obtaining a consistent non-parametric estimate, $p(Y)$, of the empirical probability of

⁷ Cohen (2008), however, presents a manageable algorithm for computing the set of PSNE for the special case where the effect of entry on profits depends on the number but not the identity of the entrants.

⁸ For a graphical representation, see Figure 1 in Tamer (2003).

observing each profile of entrants. The loss function for the MMD estimator is then defined in the following way:

$$\begin{aligned}
 & L(Y, X, \theta) = 0 && \text{if } \underline{P}(Y, X; \theta) \leq p(Y) \leq \bar{P}(Y, X; \theta) \\
 \mathbf{4.1} \quad & L(Y, X, \theta) = \|\underline{P}(Y, X; \theta) - p(Y)\| && \text{if } \underline{P}(Y, X; \theta) > p(Y) \\
 & L(Y, X, \theta) = \|p(Y) - \bar{P}(Y, X; \theta)\| && \text{if } \bar{P}(Y, X; \theta) < p(Y)
 \end{aligned}$$

where $\|\bullet\|$ is a distance measure.

The upper and lower bounds are formed in the exact same way as the case with two potential entrants, however, the calculation of the region in which a particular outcome is a unique equilibrium is much more complicated because one must check each of the possible 2^{N_m} configurations at each draw of the unobservables. As discussed above, CT reduce the number of possible outcomes by lumping similar types of firms together (for example, all small airlines are treated as one firm).

Recall that each credit union makes its outsourcing decision based on the incremental costs relative to running its own system. We assume that each firm's observed decision represents an element of a PSNE profile. This imposes the minimal condition that each firm that chooses to outsource gains a cost advantage, and no firm who provides its own system could outsource at a lower cost. These conditions are captured in equation **3.5** above.

We define a PSNE as a vector Y such that **3.5** holds for all firms in the market.

We can define the set of PSNE, $\Lambda(X; \Theta)$, as follows:

$$\mathbf{4.2} \quad \Lambda(X; \Theta) = \{Y : c_i(Y, X_i; \Theta) < 0 \Leftrightarrow y_i = 1 \quad \forall i = 1 \dots N_m\}$$

Upper and lower bounds on the probability of a particular strategy profile, Y_k are derived by calculating the probability that Y_k is a PSNE and the probability that Y_k is the *only* PSNE, respectively. In particular:

$$4.3 \quad \overline{P}_k(X; \Theta) = \Pr[Y_k \in \Lambda(X; \Theta)]$$

$$4.4 \quad \underline{P}_k(X; \Theta) = \Pr[Y_k \in \Lambda(X; \Theta) \cap |\Lambda(X; \Theta)| = 1]$$

where $|\Lambda(X; \Theta)|$ is the cardinality of $\Lambda(X; \Theta)$. The true probability must lie between these bounds. That is,

$$4.5 \quad \underline{P}_k(X; \Theta) \leq \Pr[Y_k] \leq \overline{P}_k(X; \Theta)$$

These bounds form the basis for our estimator. Following the suggestion of Manski and Tamer (2002) and CT, we form a modified minimum distance estimator by choosing the parameters that minimize the following loss function:

$$4.6 \quad \sum_{m=1}^M \sum_{k=1}^{2^{N_m}} \left[\left(\underline{P}_k(X_m; \Theta) - \Pr[Y_k] \right)^2 * I \left(\underline{P}_k(X_m; \Theta) > \Pr[Y_k] \right) + \left(\overline{P}_k(X_m; \Theta) - \Pr[Y_k] \right)^2 * I \left(\overline{P}_k(X_m; \Theta) < \Pr[Y_k] \right) \right]$$

The loss function punishes values of the parameters for which the true probabilities fall outside of the probability bounds that are generated by the model. Since the true probabilities are not observed, we replace $\Pr[Y_k]$ with a consistent estimate, $\widetilde{\Pr}[Y_k | X_m, N_m]$. The “first stage” estimator predicts outsourcing configurations as a function of the observable characteristics of each firm, observable market characteristics, observable characteristics of the other firms in the market, and the number of other firms in the market.

V. Results

We parameterize each firm's relative costs of outsourcing, from equations 3.3, as follows:

$$5.1 \quad c_i(Y) = \begin{bmatrix} \alpha + X_{im}\beta + \delta_1 K_m^{-i} + \delta_2 \log(K_m^{-i} + 1) \\ -\lambda_1 [N_m - K_m^{-i} - 1] - \lambda_2 \log(N_m - K_m^{-i}) + \varepsilon_{im} \end{bmatrix}$$

where $K_m^{-i} = \sum_{j \neq i} y_{jm}$ and X_{im} includes two market specific variables (the log of population and a dummy for rural markets) and two firm specific variables (the log of assets and the number of branches). The specification allows for the complementarities to enter both linearly and logarithmically into the costs of both outsourcing and internal IT provision.

We follow Berry's (1992) suggestion and model each firm's unobservable as a composite of firm and market level unobservables. In particular, we let

$$\varepsilon_{im} = \sqrt{1 - \rho^2} \eta_{im} + \rho \mu_m$$

where η_i is an iid standard normal firm-level unobservable, and μ_m is an iid standard normal market-level unobservable. The weighted average of the two, by construction, is standard normal. The correlation between the unobservables for firms in the same market is ρ^2 .

Our MMD estimator is given by the minimum of the following criterion function:

$$5.2 \quad \sum_{m=1}^M \left[\left(\underline{P}(Y_m | X_m; \Theta) - \widetilde{\Pr}[Y_m | X_m] \right)^2 * I \left(\underline{P}(Y_m | X_m; \Theta) > \widetilde{\Pr}[Y_m | X_m] \right) + \left(\overline{P}(Y_m | X_m; \Theta) - \widetilde{\Pr}[Y_m | X_m] \right)^2 * I \left(\overline{P}(Y_m | X_m; \Theta) < \widetilde{\Pr}[Y_m | X_m] \right) \right]$$

where Y_m is the observed outsourcing configuration for market m . $\widetilde{\Pr}[Y_m | X_m]$ is a consistent “non-structural” estimate of the probability of observing Y_m given the set of exogenous variables X_m . Finally, $\underline{P}(Y_m | X_m; \Theta)$ is the probability, derived from the structural model, that Y_m is the unique PSNE while $\overline{P}(Y_m | X_m; \Theta)$ is the probability that Y_m is a PSNE.

Specification of the first stage estimates

The most obvious way to obtain non-structural estimates of the probability of observing Y_m is to use a bin approach or a parametric multinomial qualitative response model. We do not opt for either approach for several reasons. First, we do not observe the same firms across markets. Therefore, it is not possible to define the outcomes in a way that will be comparable across markets. In addition, our decision to include markets with a large number of participants in the analysis precludes the use of a multinomial qualitative response model due to the large number of possible outcomes, 2^{16} , in the largest markets. Our solution is to begin with a qualitative response model, and then to decompose it into two manageable pieces.

We begin by considering the estimation of $\Pr(Y | X_m)$ where X_m is the matrix of exogenous variables comprised of elements that differ across firms as well as market-specific variables. We assume that $\Pr(RK_k | X_m)$ is well defined, where $RK_k \in RK$ is a particular set of rankings of firms’ relative outsourcing costs from the set RK of all possible ($N_m!$) set of rankings. This assumption allows us to decompose the probability of observing a particular outsourcing profile, Y , as follows:

$$5.3 \quad \Pr(Y | X_m) = \Pr(Y | X_m, \sum y_i = N^*) * \Pr(\sum y_i = N^* | X_m)$$

We decompose the multinomial probability on the left hand side as the multinomial probability *conditional on a particular number of outsourcers* multiplied by the probability of observing that number of outsourcers. Given our assumption that the probability of any given ranking of firms' outsourcing costs is well defined, then:

$$5.4 \quad \Pr(Y | X_m, \sum y_i = N^*) = \Pr[rk_i(X_m) < rk_j(X_m), \forall y_i = 1, y_j = 0 \in Y]$$

where rk_i is the cost ranking for firm i . We estimate **5.4** using a rank order logit whose index is a polynomial function of X_m . We estimate $\Pr(\sum y_i = N^* | X_m)$ with a Poisson regression whose mean is a polynomial function of moments of X_m , and whose distribution is right-truncated at N_m .

Computation of set of PSNE

We employ simulation methods to calculate the bounds implied by our model on the probability of observing a given outsourcing configuration, Y_k , for a given set of parameters. For a each set of draws, r , from the multivariate distribution of ε , we compute the set of PSNE, $\Lambda_r(X; \Theta)$, using the algorithm described in Appendix 2. This algorithm is feasible as the number of firms increases, and does not require one to limit the space of possible outcomes. This is due to the fact that one need not check whether each possible outsourcing configuration is a PSNE. Instead, one need only check for PSNE in the *number* of firms who outsource, which can be done generically (as described in the Appendix). Proposition 1 above implies that if there is a PSNE in which a given number of firms outsource, there must be only one such PSNE. Once the existence of a

PSNE involving a particular number of firms is verified, finding the exact equilibrium configuration simply entails ranking the firms by $X_i\beta + \varepsilon_{ir}$.

Simulation Estimators for $\underline{P}(Y_m | X_m; \Theta)$ and $\overline{P}(Y_m | X_m; \Theta)$

The MMD estimator in 5.2 requires one to compute the probability that the outcome Y_m is (1) the unique PSNE, $\underline{P}(Y_m | X_m; \Theta)$, and (2) a PSNE, $\overline{P}(Y_m | X_m; \Theta)$. We use an importance sampling technique that takes quasi-random draws from the distribution of μ_m , the market-specific unobservable. For each draw, d we find the set of firm-specific unobservables, A_d , that is consistent with the observed outcome. That is,

$$5.5 \quad A^d = \left\{ \eta_m^{-d} : Y_m \in \Lambda \mid \mu_m^d \right\}$$

A^d is an N_m dimensional rectangle with endpoints:

$$\left[-\infty, \frac{\alpha + X_{im}\beta + \delta_1 K_m^{-i} + \delta_2 \log(K_m^{-i} + 1) - \lambda_1 [N_m - K_m^{-i} - 1] - \lambda_2 \log(N_m - K_m^{-i}) + \rho \tilde{\mu}_m^d}{-\sqrt{1 - \rho^2}} \right] \text{ if } y_{im} = 1$$

and,

$$\left[\frac{\alpha + X_{im}\beta + \delta_1 K_m^{-i} + \delta_2 \log(K_m^{-i} + 1) - \lambda_1 [N_m - K_m^{-i} - 1] - \lambda_2 \log(N_m - K_m^{-i}) + \rho \tilde{\mu}_m^d}{-\sqrt{1 - \rho^2}}, \infty \right] \text{ if } y_{im} = 0.$$

Since the firm-specific unobservables are iid and independent of the market specific unobservable, one can, for each simulated draw $\tilde{\mu}_m^d$, easily (1) compute analytically the probability that a given outcome is a PSNE and (2) take random draws from the set A^d .

The simulation estimator for the upper bound on the likelihood of observing Y_m is:

$$5.6 \quad \tilde{P}(Y_m | X_m; \Theta) = \frac{1}{D} \sum_{d=1}^D \Pr(\bar{\eta}_m \in A_d)$$

The simulation estimator for the lower bound on the likelihood of observing Y_m is:

$$5.7 \quad \underline{P}(Y_m | X_m; \Theta) = \frac{1}{D} \sum_{d=1}^D I\left(\left|\Lambda\left(\bar{\eta}_{im}^d \in A^d\right)\right|=1\right) * \Pr(\bar{\eta}_m \in A^d)$$

The independence of η_{im} and μ_m allows us to take a single draw from A^d for each $\bar{\mu}_m^d$.

Parameter estimates

Table 4 presents the values of the parameters that minimize the MMD criterion function in 5.2.

For purposes of comparison, the right hand panel presents the estimates we would have obtained had we estimated each firm's cost minimization problem as a probit ignoring the existence of multiple PSNE. We also ignore correlation among the errors of firms in the same market, though this correlation is estimated to be quite small in the structural model.

Complementarities appear to be present in both outsourcing and internal IT provision. That is, firms' costs appear to decrease to the extent that additional firms are adopting the same technology. The complementarities for internal IT provision are increasing at a decreasing rate (with a non-trivial effect on the log term), while

complementarities for outsourcing appear to be linear (with the log term almost equal to zero).

In general, the MMD will identify the set of parameters consistent with the behavioral model, but the parameters may not be point identified. Chernozhukov, Hong and Tamer (2003) provide a method for calculating the bounds on the parameters that cover the identified set with the desired probability. Appendix 3 discusses identification of the model. We provide different sets of sufficient conditions for point identification, depending on the parametric specification. Assuming that these conditions are satisfied allows us to use bootstrap techniques to calculate standard errors which will generally be tighter than the bounds discussed in Chernozhukov, Hong and Tamer. STANDARD ERRORS TO BE COMPUTED XXX.

Measures of Coordination Failure

The power of our modeling approach can be seen as we attempt to measure the extent of coordination among credit unions in each market. Because our model is structural, we can simulate equilibrium outcomes. Because we do not impose a selection rule in the presence of multiple PSNE, we can compare observed outcomes to other PSNE outcomes consistent with the observed outcome to address which PSNE are selected.

Table 5 characterizes the observed outcomes in terms of the likelihood of multiplicity and the cost characteristics of alternative PSNE when they are present. All of our exercises condition on the observed outcome being a PSNE. That is, we integrate over the region of the error terms, η_{im} and μ_m , that are consistent with Y_m being an

element of Λ , the set of PSNE. The first column of Table 5 presents the probability that there are other PSNE, given that the observed outcome is a PSNE, i.e.,

$$5.8 \quad \Pr(|\Lambda| > 1 | Y_m \in \Lambda)$$

As one would expect, the prevalence of multiple PSNE increases with the number of firms in the market.

The second column of table 5 presents the probability of a lower total cost PSNE, given the existence of multiple PSNE, i.e.,

$$5.9 \quad \Pr\left(\sum \alpha_{im}(Y'_m \in \Lambda) - \sum \alpha_{im}(Y_m) < 0 \mid Y_m, Y'_m \neq Y_m \in \Lambda \cap |\Lambda| > 1\right)$$

where $\alpha_{im}(Y_m)$ captures each firm's realized IT cost, in outsourcing configuration Y_m .

$\alpha_{im}(Y_m)$ is not identified in our model, however, $\alpha_{im}(Y'_m \in \Lambda) - \alpha_{im}(Y_m \in \Lambda)$ is identified.

Once again, as the number of firms in the market increases so do PSNE that would reduce the *total* costs. This should not necessarily be construed as a coordination failure, since some firms may be better off in an alternative PSNE while others might be worse off. The third column presents the number of firms who would be better off (i.e., incur lower costs) in the least total cost equilibrium, assuming that it is not the observed outcome, i.e.,

$$5.10 \quad E \left(\sum I(\alpha_{im}(Y'_m \in \Lambda) - \alpha_{im}(Y_m) < 0) \mid Y_m, Y'_m \in \Lambda \cap \sum \alpha_{im}(Y'_m \in \Lambda) - \sum \alpha_{im}(Y_m) < 0 \right)$$

The idea here is that markets with lots of firms that would be better off in the lower total cost PSNE are, on average, less coordinated than those markets with fewer firms that would be better off in the least cost equilibrium. To obtain a measure that will allow us to compare markets with different numbers of firms, we compute the expected proportion of

firms that would be better off in the least cost PSNE, when one exists. This measure, which is simply **5.10** divided by the total number of firms, is presented in the fourth column. Here we observe the U-shaped pattern relating coordination to the number of firms in the market. Coordination gets progressively stronger, with a lower proportion of firms better off in the least total cost PSNE, as one goes up to five firms, at which point it gets progressively weaker looking at markets with two firms up to five firms, and then weaker moving above five firms.

A more definitive measure of coordination failure is to examine the likelihood that the observed outcome is pareto dominated by another PSNE, given that multiple PSNE exist. This measure is presented in the far right column of table 5. Once again, a U-shaped pattern of coordination failure is observed, with the minimum being at five firms. Figure 1 plots our two measures of coordination failure against the number of firms in our observed markets.

VI. Conclusion

This paper extends the literature on the estimation of game theoretic models of firm behavior in several dimensions. Our example of credit unions' outsourcing decisions is a game with strategic complementarities and multiple PSNE. It represents an ideal laboratory for estimating complementarities since credit unions are known to communicate much more than firms that compete directly with one another.

We show how to apply techniques developed for the estimation of entry models with multiple PSNE to our problem. Our algorithm for finding the fixed points of firms' best reply correspondences uses prior information about the properties of the set of PSNE

to substantially alleviate our computational burden. In addition, we show that the methods developed by CT and ABJ can be generalized to games with many players where one does not observe the same set of agents across markets.

The fact that our model does not impose any equilibrium selection rule when multiple PSNE are present allows us to characterize the degree of coordination in the observed outcomes. Our main empirical finding is the U-shaped relationship between measures of coordination failure and the number of firms.

Appendix 1: Proof of Propositions

Proposition 1: Any PSNE in which N firms choose to outsource, will always involve the same N firms choosing to outsource.

Proof:

Suppose not. Then there must exist at least one firm such that: $c_i(Y) < 0$, $c_i(Y') > 0$ where $\sum y_j = \sum y'_j$, and both Y and Y' are NE. Since firm i outsources in profile Y , its incremental costs are

$$c_i(Y) = X_i\beta + f\left(\sum_{j \neq i} y_j\right) < 0 \text{ by the fact that } Y \text{ is a NE.}$$

In configuration Y' , i does not outsource. Its incremental costs, if it were to outsource, must be positive if Y' is a NE. Those costs are given by:

$$c_i(Y') = X_i\beta + f\left(\sum_{j \neq i} y'_j\right) = X_i\beta + f\left(\sum_{j \neq i} y_j + 1\right) = c_i(Y) + f\left(\sum_{j \neq i} y_j + 1\right) - f\left(\sum_{j \neq i} y_j\right)$$

But $c_i(Y')$ must be negative because $c_i(Y) < 0$ and f is weakly decreasing. This contradicts the supposition that Y' is a NE.

QED.

Proposition 2: The N firms with the *lowest* incremental costs of outsourcing will all choose to outsource in any pure strategy Nash equilibrium in which N firms outsource.

Proof:

Since the outsourcing decision of a firm affects all other firms' costs equally, the ranking of incremental costs does not depend on how many other firms choose to outsource. That is,

$$c_i(Y_N) < c_j(Y_N) \Leftrightarrow c_i(Y'_N) < c_j(Y'_N).$$

This also implies that

$$c_i(Y_N) < c_j(Y_N) \Leftrightarrow X_i\beta + \varepsilon_i < X_j\beta + \varepsilon_j$$

Therefore, it is possible, without loss of generality, to rank firms' incremental costs in the following way: $c_1 < c_2 < \dots < c_{N_m}$.⁹

⁹ The error terms insure that ties occur with zero probability.

In an equilibrium in which N firms outsource, there must be N firms with $c_i \leq 0$ and $N_m - N$ firms with $c_j > 0$.

By proposition 1, there must be *only* one set of N firms such that $c_i \leq 0$.

Therefore, in a NE with N firms outsourcing, it must be the N firms with the lowest incremental cost that choose to outsource.

That is, if Y is a NE, then $c_1(Y_N) < \dots < c_N(Y_N) < 0 < c_{N+1}(Y_N) < \dots < c_{N_m}(Y_N)$ **QED.**

Proposition 3: If there is a PSNE in which N firms choose to outsource, a PSNE in which $N + 1$ firms choose to outsource does not exist.

Proof:

Suppose Y_N is a NE profile in which N firms outsource. By Proposition 2, it must be possible to arrange firms by their incremental costs in the following way.

$$c_1(Y_N) < \dots < c_N(Y_N) < 0 < c_{N+1}(Y_N) < \dots < c_{N_m}(Y_N)$$

The fact that the complementarity in the cost of outsourcing depends only on the decisions of the other firms in the market, implies that $c_{N+1}(Y_N) = c_{N+1}(Y_{N+1})$

Therefore, $c_{N+1}(Y_{N+1}) > 0$ and Y_{N+1} cannot be a NE.

QED.

Appendix 2: Algorithm to compute set of PSNE outsourcing configurations

(1) For a given market m , calculate the matrix of incremental outsourcing costs, $C^d(\Theta)$, for a particular draw, d , for all firms under each possible *number* of outsourcers. The element corresponding to the i^{th} row and t^{th} column, c_{it}^d (suppressing the dependence on the parameters), represents the incremental outsourcing costs for firm i assuming that $t-1$ other firms outsource. That is,

$$c_{it}^d = X_i \beta + \delta_1 (t-1) + \delta_2 \log(t) - \lambda_1 (N_m - t) - \lambda_2 \log(N_m - t + 1) + \varepsilon_{id} \text{ for } t = 1, \dots, N_m$$

(2) Calculate the matrix of firms' best replies, at each draw, to $t-1$ firms outsourcing, $R^d(\Theta)$ as follows

$$\begin{aligned} r_{it}^d &= 1 \text{ if } c_{it}^d < 0 \\ r_{it}^d &= 0 \text{ if } c_{it}^d \geq 0 \end{aligned}$$

(3) Calculate the number of firms in every PSNE.

Let $R_{(t)}^d$ be the t^{th} column of $R^d(\Theta)$.

$R_{(1)}^d$, i.e., the profile in which no firms outsource, is a PSNE if $\sum_{i=1}^{N_m} r_{i1}^d = 0$. That is, each firm's outsourcing costs are computed assuming no other firms choose to outsource. If, given this assumption, it is not optimal for any firm to outsource, then $R_{(1)}^d \in \Lambda^d(X; \Theta)$.

For configurations in which at least one firm chooses to outsource, the following two conditions are necessary and sufficient to conclude that $R_{(t)}^d$ is a PSNE, i.e.,

$$R_{(t)}^d \in \Lambda^d(X; \Theta).$$

$$\mathbf{C1:} \quad \sum_{i=1}^{N_m} r_{i,t}^d = t$$

$$\mathbf{C2:} \quad \sum_{i=1}^{N_m} r_{i,t+1}^d = t$$

The first condition says that there are exactly t firms who find it optimal to outsource given that $t-1$ other firms choose to outsource. The second condition says that the $N_m - t$ firms who (if **C1** is satisfied) would not outsource given that $t-1$ other firms outsource, also would not outsource given that t other firms choose to outsource.

Therefore, the set of PSNE for a given draw are given by t^{th} column of $R^d(\Theta)$ for: (1) all $t > 1$ that satisfy **C1** and **C2** and (2) $t = 1$ if **C1** is satisfied. It is possible to perform a directed search for the fixed points of the best reply correspondence in this manner because Proposition 1 implies that there must be only one PSNE configuration for a given number of outsourcers

Appendix 3: Identification

We discuss identification for two different parameterizations of the relative costs of outsourcing. The first is a strictly linear parameterization which is more restrictive than the specification that we estimate from equation 5.1. The second is a fully flexible parameterization that is less restrictive than our specification, but requires stronger assumptions.

Linear Parameterization

The bounds in equation 4.5 are the basis for the objective function that we minimize to obtain parameter estimates. As discussed in CT, there may be a set of parameters, rather than a single parameter vector, that exactly satisfy these bounds in the population. Whether the model is point identified will depend on whether a sufficient number of the bounds defined in 4.5 collapse to a point, so that the parameter vector that satisfies the bounds in the population is unique. That is, one must determine whether there are a sufficient number of equality restrictions in 4.5.

The logic of the identification argument follows from Tamer (2003), with two principle differences. First, we are interested in identifying the effect of the number firms in the market who outsource separately from the effect of the number of firms who provide IT in-house. Second, and crucial to our arguments for separately identifying the outsourcing and in-house effects, we observe some markets where there is only one credit union. We begin by discussing identification in the straightforward case which assumes that complementarities enter costs linearly. We then provide a brief discussion of identification using more flexible functional forms.

Assume that the incremental costs associated with outsourcing take the following form:

$$\mathbf{A3.1} \quad c_i(Y) = \alpha + X_{im}\beta + \delta K_m^{-i} - \lambda [N_m - K_m^{-i} - 1] + \varepsilon_{im}$$

where K_m^{-i} is the number of firms in the market, excluding firm i , that outsource, and $N_m - K_m^{-i} - 1$ is the number of firms other than i that provide IT in-house. For one-firm markets, 4.7 becomes:

$$\mathbf{A3.2} \quad c_i(Y) = \alpha + X_{im}\beta + \varepsilon_{im}$$

In this case, a firm's outsourcing decision depends only on the exogenous variables in X and the likelihood is well defined so that α and β are identified (assuming sufficient variation in X). For two firm markets, the bounds in 4.5 are as follows:

$$\begin{aligned}
 \text{A3.3} \quad & \underline{\Pr}_{0,0}(X; \Theta) < \widetilde{\Pr}[\{0,0\} | X] < \overline{\Pr}_{0,0}(X; \Theta) \\
 & \underline{\Pr}_{1,0}(X; \Theta) < \widetilde{\Pr}[\{1,0\} | X] < \overline{\Pr}_{1,0}(X; \Theta) \\
 & \underline{\Pr}_{0,1}(X; \Theta) < \widetilde{\Pr}[\{0,1\} | X] < \overline{\Pr}_{0,1}(X; \Theta) \\
 & \underline{\Pr}_{1,1}(X; \Theta) < \widetilde{\Pr}[\{1,1\} | X] < \overline{\Pr}_{1,1}(X; \Theta)
 \end{aligned}$$

If either (1,0) or (0,1) is a PSNE, it must be the only PSNE since there can be only one equilibrium in which one firm outsources and there cannot be another PSNE in which either no firms or two firms outsource by Propositions 1 and 3, respectively. Therefore, the upper and lower bounds on both (1,0) and (0,1) must be the same. The equality restrictions associated with these two outcomes are as follows:

$$\begin{aligned}
 \text{A3.4} \quad & \Pr(\alpha + X_1\beta + \lambda < 0, \alpha + X_2\beta + \delta > 0) = \widetilde{\Pr}[\{1,0\} | X] \\
 & \Pr(\alpha + X_1\beta + \delta > 0, \alpha + X_2\beta + \lambda < 0) = \widetilde{\Pr}[\{0,1\} | X]
 \end{aligned}$$

The unique outcomes and the equality restrictions they imply are sufficient to identify $\alpha + \lambda$ and $\alpha + \delta$. Separately identifying λ and δ is possible because α is separately identified by the one firm markets.

In the population, therefore, it would be sufficient to observe only the one and two firm markets to identify the parameters of interest. In our sample, however, markets with more than two firms as well as the non-unique outcomes from two firm markets provide additional information through the bounds and thereby assist in parameter inference.

Flexible parameterization

One might be interested in estimating a more flexible specification of the effect of strategic complementarities on the incremental costs of outsourcing such as:

$$\text{A3.5} \quad c_i(Y) = \alpha + X_{im}\beta + \delta_{K_m^{-i}} + \lambda_{N-K_m^{-i}-1} + \varepsilon_{im}$$

where $\delta_{K_m^{-i}}$ and $\lambda_{N-K_m^{-i}-1}$ are different shift parameters for each possible number of firms that outsource and provide IT in-house, respectively. In this case, point identification will be facilitated if one assumes that one of the firm-specific variables in X has infinite support and empirical content (in the sense of having a non-zero coefficient).

The full support assumption guarantees that there is a positive density associated with outcomes in which any number of firms' decisions are predetermined by an infinitely large or small value of $X\beta$ (this is the same logic as Theorem 1 in Tamer (2003)). Identifying δ_1 and λ_1 would proceed in the same manner that δ and λ were identified in the linear case, using two firm markets. δ_2 is identified from observations in three firm markets for which two of the firms' X 's are infinitely large (for an X -variable that reduces the incremental cost of outsourcing). The full support assumption guarantees that there is a positive density associated with this type of market in the population. Then, for any error vector, the two firms will always choose to outsource and the only PSNE will be (2,1) and (3,0), which are mutually exclusive outcomes allowing one to identify δ_2 . Identification of λ_2 is achieved similarly, and then a similar argument can be made to show identification of δ_3 and λ_3 and so forth.

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TABLE 1: Outsourcing Configurations, By Number of Firms in Each Market

# Outsourcers	Number of Firms in Market															
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
0	82%	61%	44%	56%	33%	27%	25%	17%	9%	33%	27%	-	7%	7%	11%	8%
1	18%	31%	41%	27%	38%	30%	34%	13%	18%	27%	13%	17%	7%	-	22%	8%
2		8%	12%	12%	13%	16%	13%	17%	23%	13%	27%	50%	14%	20%	11%	8%
3			3%	5%	8%	16%	16%	17%	18%	13%	13%	-	-	20%	-	17%
4				-	10%	8%	3%	22%	23%	-	7%	-	43%	27%	11%	17%
5					-	3%	9%	-	5%	-	7%	-	7%	7%	11%	-
6						-	-	-	5%	-	7%	-	7%	7%	-	17%
7							-	13%	-	13%	-	17%	-	7%	-	8%
8								-	-	-	-	-	-	-	22%	-
9									-	-	-	17%	7%	7%	-	-
10										-	-	-	-	-	-	-
11											-	-	-	-	11%	8%
12												-	-	-	-	-
13													7%	-	-	-
14														-	-	8%
15															-	-
16																-
Total # Markets	502	202	98	59	40	37	32	23	22	15	15	6	14	15	9	12

TABLE 2: Summary Statistics for Explanatory Variables

	Mean	Std Dev	Min	Max
Market Level Variables				
Number of Markets=1101				
Population	80,278	138,269	546	1,722,256
Log of Population	10.62	1.12	6.30	14.36
Dummy for Rural Markets	.82	.39	0	1
Firm-Level Variables				
Number of Firms=3570				
Assets (in 000s)	45,413	131,052	83	2,563,731
Log of (Assets/1000)	9.28	1.71	4.42	14.76
Size of Branch Network	1.77	2.00	1	36

Note: Logs of Population and Assets/1000 used in all estimated models

Table 3: Estimates from Baseline Models

Dependent Variable = Probability of outsourcing

	Probit Model		Linear Probability Model	Instrumental Variables Linear Probability Model
	<i>Estimates</i>	<i>Marg. Effects</i>		
# of other firms who outsource	.1569 (.0119)	.0456 (.0035)	.0492 (.0033)	.0421 (.0075)
# of other firms who insource	-.0370 (.0093)	-.0108 (.0027)	-.0101 (.0025)	-.0063 (.0044)
Log of Assets	.3488 (.0214)	.1014 (.0059)	.0812 (.0050)	.0825 (.0051)
Branch Network	-.2500 (.0214)	-.0726 (.0061)	-.0499 (.0041)	-.0499 (.0041)
Log of Population	-.0814 (.0367)	-.0237 (.0107)	-.0209 (.0101)	-.0210 (.0101)
Dummy Variable for Rural Markets	-.0279 (.0787)	-.0081 (.0229)	-.0015 (.0216)	.0067 (.0230)
Constant	-2.7418 (.4376)		-.2149 (.1182)	-.2345 (.1198)
R-squared	0.1405		0.1425	0.1406
N=3570				
Standard Errors in Parentheses				

Table 4: Estimates from Full Model

Dependent Variable = Incremental Costs of Outsourcing

	<u>Modified Minimum Distance Estimator</u>	<u>Probit</u>	
	Estimates	Estimates	Std. Errors
# of other firms who outsource	-0.286	-0.069	0.030
Log(# other firms that outsource+1)	-0.002	-0.326	0.102
# of other firms who insource	-0.041	-0.035	0.020
Log(# other firms that insource+1)	-0.394	-0.039	0.090
Log of Assets	-0.302	-0.349	0.021
Branch Network	0.137	0.250	0.022
Log of Population	0.075	0.080	0.038
Dummy Variable for Rural Markets	0.032	-0.008	0.076
Constant	2.257	2.825	0.442
Rho ² (correlation between market and firm-specific errors)	0.003		

N=3570

Standard errors for structural model to be computed

Note: Since the dependent variable is the incremental **cost** of outsourcing, lower values correspond to a greater probability of outsourcing

Table 5: Characteristics of Equilibria given that Observed Outcome is an Equilibrium

# Firms	Prob(Multiple PSNE, given observed outcome is a PSNE)	Prob of equil w/lower total costs, given multiple PSNE	Expected # of firms better off in least total cost PSNE, given existence of lower total cost PSNE	Expected proportion of firms that would be better off in least total cost PSNE, given existence of lower total cost PSNE	Prob that observed is pareto dominated, given existence of multiple PSNE
2	5.5%	46.1%	1.47	0.73	22.5%
3	11.8%	63.5%	2.03	0.68	14.6%
4	19.5%	62.6%	2.65	0.66	8.2%
5	33.8%	66.5%	3.32	0.66	5.8%
6	50.9%	79.7%	4.09	0.68	6.9%
7	62.7%	84.1%	4.87	0.70	7.4%
8	83.9%	83.5%	5.81	0.73	8.5%
9	89.9%	97.1%	6.94	0.77	11.8%
10	94.3%	95.8%	8.04	0.80	14.8%
11	98.6%	99.3%	9.24	0.84	18.7%
12	99.9%	98.5%	10.91	0.91	33.2%
13	99.3%	93.4%	11.23	0.86	37.0%
14	100.0%	100.0%	13.34	0.95	53.8%
15	100.0%	99.9%	14.37	0.96	53.9%
16	100.0%	98.5%	15.68	0.98	72.1%

Figure 1: Measures of Coordination Failure

